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LATITUDE AND LONGITUDE, AND THE SECULAR
MOTION OF THE POLE

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I. INTRODUCTION

Astronomical evidence has been used in the study of two questions which concern geophysics and geology. These are the secular motion of the pole and continental drift. Astronomical

theory cannot provide information on these matters for the remote past or for the distant future. Astronomical observation, however, can provide determinations of astronomical latitude and longitude of stations for different epochs. An analysis of the results provides indicated changes in latitude and longitude. It does not necessarily follow, however, that if changes in latitude and longitude are found the stations have moved. The changes may be due to errors of observation.

A comparison of the latitudes and longitudes obtained in this century with the values obtained about 1860 and 1870 shows large changes for a number of field stations. The indicated shift of Greenland in about 60 years is $5^{\circ} - 7.5^{\circ}$ to the west and $3''$ to the south. The large change in longitude can be ascribed to the relatively crude longitude determinations which were necessarily made before the introduction of radio time signals. The change in latitude is explained by the large uncertainty in the method of observation used in Greenland in 1870. The indicated shifts in position were also within the errors of observation in other similar cases studied.¹

It is known that horizontal and vertical crustal displacements of small magnitude do occur. The best known example of relative horizontal displacement in modern times has occurred in the region of the San Andreas fault in California. Triangulation measurements made from 1880 to 1947 show relative displacements of about 10 ft between the two sides of the fault.² Small vertical displacements, notably in Finland, have been measured by levelling operations.³ The indicated uplift for Finland from 1900 to 1950 is about 1.5 ft.

Triangulation surveys made at different epochs indicate that horizontal displacements for stations not located near faults are very small. Since 1 ft at the surface of the earth equals about $0''.01$ it is evident that the most refined astronomical methods must be employed in the study of such shifts.

In 1899, what is now the International Association of Geodesy established a chain of six latitude stations on the parallel of latitude $+39^{\circ}$ to determine the motion of the pole. A complete northern chain and a partial chain in the southern hemisphere form the International Latitude Service (I.L.S.). Observations for latitude have also been made at a number of independent stations.

It is the results of the northern I.L.S. stations, however, that are of principal importance in studying the secular motion of the pole. The reason is, the position of the pole derived from results of a chain of three or more stations is independent of errors in positions and proper motions of the stars observed, if the same stars are observed at all stations on each night. The same stars are not always observed at all I.L.S. stations because of weather conditions. The positions of the stars are corrected internally, so that asymmetry of observation has only a small second order effect on the derived position of the pole. For independent stations, on the other hand, these errors enter directly in the results, and a progressive change in latitude cannot be distinguished from a systematic error in proper motion.

Analyses of the polar motion made about 20 years after the I.L.S. began operations indicated that the centre about which the periodic motion took place was moving in the direction of North America at a rate of about $0''.003$ to $0''.006$ yr. This apparent motion could be due to three causes: (a) secular motion of the pole, (b) horizontal displacements of individual stations, and (c) asymmetry of observation.

It was recognized that there were not enough results available to distinguish between the three possibilities. The total drift in the centre of motion from 1900 to 1920 was only about $0''.08$. Hence, various investigators reached different conclusions, depending upon what other considerations, such as geophysical, were taken into account.

It was suggested in 1922 that the apparent secular motion of the pole was due to a horizontal displacement to the south of the I.L.S. station at Mizusawa, Japan. The view that Mizusawa had moved became widely accepted.

The mean position of the instantaneous pole for an interval of about six years, called the *mean pole*, is practically free of the 12- and 14-month variations. It was noticed in 1922 that the motion of the mean pole was not in a straight line, but that seemingly sudden changes in direction occurred. This phenomenon was well marked by 1954.

Although much observational material was available by that time, it was still not possible to decide if there was a real wandering of the pole because of inhomogeneities which had been introduced

into the I.L.S. results. The general view of astronomers, based on the results of the I.L.S. and of independent stations, was that the existence of a secular motion of the pole had not been proved.

In 1959 a publication of the Central Bureau of the I.L.S. appeared which removed one of the principal sources of uncertainty.⁴ By this time nearly 60 years of observation were available, as were improved fundamental systems of star positions and proper motions. The total change in position of the centre of motion of the pole since 1900 was then about $0''\cdot2$. It therefore appeared desirable to make a new study of the polar motion.

This study showed that the motion of the mean pole from 1900 to 1959 was not at random, but can be well represented mathematically as the sum of a secular linear motion and a libration of period 24 years. The various I.L.S. stations appear to be fixed relative to each other, and Mizusawa gives the least indication of any of undergoing horizontal displacements. It appears unlikely that asymmetry in the I.L.S. observations can account for the observed motion of the mean pole.

A study of the latitudes and longitudes of independent stations, although of much less weight than that of the I.L.S. results, indicates a secular motion which is in accord with that derived from the I.L.S. observations.

There thus appears to be a strong probability that the secular motion and libration found for 1900 to 1959 are real. However, no physical causes for these motions are known, and there is no assurance that these motions will continue. Future results of the I.L.S. and of independent stations will be awaited with interest.

The discussion of the secular motion of the pole is preceded by a description of astronomical methods of determining latitude and longitude.

II. ASTRONOMICAL TECHNIQUES

1. Rotation of the Earth

The earth rotates about an instantaneous axis, denoted L . Because of the gravitational attraction of the moon and sun on the equatorial bulge of the earth, the orientation of L in space changes. The progressive part of this motion is called *precession* and the

periodic parts are collectively called *nutation*. The principal term in nutation has a period of 18·6 years.

The axis of figure of the earth is defined as the principal axis of inertia. Let P and F , respectively, be the points where the axis of rotation and axis of figure intersect the surface of the earth, near the north pole. Then, according to dynamical theory, P can move about F . The observed period varies, but is about 14 months. The motion is roughly circular. The radius varies from about 15 to 35 ft.

The term *motion of the pole*, by common usage, refers to the motion of P with respect to a point fixed on the earth and not to the motion of L in space. The polar motion has two well known periodic components, of periods 12 and 14 months. Unlike precession and nutation, these terms are not due to gravitation.

2. Equatorial System, Astronomical

L pierces the celestial sphere in two points N and N' , which are respectively the north and south *celestial poles*. The great circle which is everywhere 90° from these points is the *celestial equator*. *Declination*, δ , is the angular distance of a star from the equator. The north polar distance is $\beta = 90^\circ - \delta$.

Because of the rotation of the earth, a star appears to move about N , diurnally, in a circle of radius β . Thus, δ may be found in a fundamental manner by measuring the diameter of the diurnal circle. A photograph of circumpolar star trails could, in theory, furnish fundamental declinations.

In practice, fundamental declinations are obtained with a meridian transit circle. This is a telescope which has only one degree of freedom; its optical axis may be directed to any point in the meridian. An essential part of the transit circle is an accurately divided circle which enables arcs along the meridian to be measured. Fundamental declinations of circumpolar stars are obtained by measuring the altitudes at transit above pole and below pole. The fundamental declinations of other stars may be obtained by transfer from circumpolar stars.

The circle reading of the nadir is found by pointing the instrument downward at a basin of mercury so that the micrometer threads are seen to coincide with the reflected image. The altitude

of a star may be determined from its circle reading at time of transit and the circle reading of the nadir.

Let A_1 and A_2 be the altitudes of a circumpolar star as observed above pole and below pole, respectively, 12 h apart. Then

$$\delta = 90^\circ - (A_1 + A_2)/2$$

which is independent of the latitude of the station.

The computation of A_1 and A_2 requires that corrections shall be made for refraction, instrumental flexure, and division errors of the circle. For stars near declination 90° , the difference in corrections for A_1 and A_2 are small. The position of the equator may be found by observing the sun and minor planets. Hence, fundamental declinations are found with the highest accuracy for stars near 0° or 90° declination.

It may be noted that the motion of the pole, as defined above, does not affect the declination of a star. The motion of L , however, does affect it. The declination will change, also, if the star has proper motion. Hence, the difference in declination of a star at two epochs is due to precession, nutation, and proper motion. The first two may be obtained by calculation, and proper motion may then be derived. The proper motions of stars with declinations from about 20° to 60° are determined with less accuracy than those near the equator or pole.

The sun appears to move in a plane called the *ecliptic*. The *vernal equinox* is the intersection between the ecliptic and the equator, at which the declination of the sun increases.

Right ascension is defined as the angle between the great circles which pass respectively through the vernal equinox and a star, measured eastward.

Observations of the sun provide the position of the equinox and the inclination of the equator to the ecliptic. Having these, further observations of the sun will provide its fundamental declination and right ascension. By observing the sun and stars, and through use of an accurate clock, fundamental right ascensions of stars may be obtained. Fundamental proper motions in right ascension may be determined in a manner analogous to that described for declination.

Observations made over intervals of about five or six years are combined and published in the form of observational catalogues,

From a series of such catalogues, extending over intervals of 50 to 100 years, there may be derived positions for the mean epoch and also proper motions. These constitute a fundamental system, which is published as a fundamental catalogue.

Generally speaking, observations made in the eighteenth and nineteenth centuries are affected by systematic errors which are much larger than those of observations made in the twentieth century. As observations accumulated it became possible to increase the precision of fundamental catalogues by dropping off the oldest series of fundamental observations. The fundamental catalogues known as *FK3* and *G.C.* have mean epochs of about 1900.^{5,6} The catalogue *N30* has a mean epoch of about 1930.⁷ It is based on observational catalogues whose mean epochs are from 1917 to 1949. *FK3* will be replaced by an improved catalogue, *FK4*, about 1962.

The gradual increase in accuracy of fundamental catalogues means that secular changes in latitude may be determined with increased precision as time goes on. At present, in 1959, systematic errors in proper motions of the order of several thousandths of a second of arc per year may exist in even the most modern catalogues.

3. Astronomical Latitude and Longitude

Positions on the earth may be designated by giving the astronomical latitude and longitude. These coordinates cannot be used directly to furnish the geometric position of a station with respect to the centre of mass of the earth because of the existence of gravity anomalies. However, they may be determined with high precision and are therefore useful for studying displacements of stations and the motion of the pole.

The *instantaneous astronomical latitude*, ϕ , of a station, S , is the complement of the angle between the vertical at S and a line L , through S which is parallel to the instantaneous axis of rotation, L .

Let Q_s be the plane which contains the vertical and L_s . Let Q_a be the plane which passes through the vertical and a line parallel to L at the Airy transit circle of Greenwich, at the original site. The angle between Q_s and Q_a is the *instantaneous astronomical longitude* of S .

It may be shown that the latitude is equal to the altitude of the celestial pole at a station.

4. Determination of Latitude

Latitude may be determined by observing the zenith distance, ζ , of a star of known declination. If the observation is made in the meridian we have $\phi = \delta - \zeta$, where ζ is reckoned positive to the north. Zenith distance may be measured with the aid of a graduated circle and a spirit level.

Modern field instruments of the most precise type utilize circles engraved on glass. Despite the small size of the circle, about 3 in. diameter, the readings may be made to about 1" in some theodolites.

The accurate determinations of latitude at fixed stations, however, utilize methods which avoid the use of graduated circles entirely.

A. Horrebow Transit Method

A suitable pair of stars of known declination δ_1 and δ_2 , respectively, is observed at nearly the same altitude in the meridian, one to the north and the other to the south. The telescope is reversed about a vertical axis between the observations. The inclination of the optical axis to the vertical is kept the same in the two positions by means of a spirit level. Micrometer settings are made on each star as it trails through the field of view. The latitude is

$$\phi = \delta_1 - \delta_2 - (r_1 - r_2)/2$$

where $(r_1 - r_2)$ is the difference in micrometer readings converted into arc.

B. Photographic Zenith Tube (PZT) Method

This method is a form of the preceding one. The same star is photographed near the zenith with the lens and plate rotated as a unit, several times, through 180° . Hence, $\delta_1 = \delta_2$. The measurement of $(r_1 - r_2)$ is made with a measuring engine. The local sidereal time is determined simultaneously.

C. Danson Astrolabe Method

This instrument is operated in conjunction with a clock. The instant is determined by observation when the zenith distance of a star is 30° . By observing a number of stars of known right ascension and declination in various azimuths it is possible to determine the latitude and local sidereal time.

In all of the three methods described the observations, essentially, are made at equal altitudes on opposite sides of the zenith. If the refraction depends only upon altitude and not upon azimuth the results are independent of refraction.

The latitude obtained is directly dependent upon the declinations used. A systematic error in the proper motions will cause a spurious, progressive change in latitude. There are two ways of avoiding this effect. One is to use a chain of stations on the same parallel. The other is to utilize observations of circumpolar stars. It can readily be shown that

$$\phi = (\delta_1 + \delta_2)/2$$

which is independent of the declination of the star observed.

It has been frequently suggested that circumpolar transit observations be used to study secular changes in latitude. The transit circles are designed, however, for the determination of declinations and right ascensions. Latitude cannot be accurately determined because it cannot be well separated from other unknowns, such as refraction, flexure, and division errors. For this reason, the latitude determined as a by-product through observations of circumpolars is not considered by astronomers as having high precision.

Concurrent determinations of latitude obtained with the 6 in. and 9 in. transit circles of the U.S. Naval Observatory at Washington illustrate the uncertainty involved. C. B. Watts, F. P. Scott, A. N. Adams, and G. van Herk have provided me with the following determinations of latitude:

Interval	Latitude (6 in.)	Interval	Latitude (9 in.)
1911-18	$38^{\circ} 55' 13''\cdot73$	1903-13	$38^{\circ} 55' 14''\cdot07$
1925-30	13-86	1913-18	13-91
1930-35	13-77	1918-25	14-07
1936-41	13-86	1935-45	13-73
1941-49	13-91		

Least square solutions indicate shifts in latitude of $+0''\cdot006$ yr for the 6 in. and $-0''\cdot010$ yr for the 9 in. The indicated relative shift in latitude in 40 years is about $0''\cdot5$, or 50 ft. With the two instruments located side by side, 164 ft apart, no such displacement occurred. Thus, we cannot rely on circumpolar observations for precise information on latitude changes.

5. Longitude

The difference in longitude between two stations is obtained by determining the local times and measuring the difference with the aid of radio time signals.

The most accurate method of monitoring time signals is by use of the cathode ray oscilloscope. The incoming signal is displayed on the oscilloscope, along with time markers produced by a local clock. The leading edge of the signal may be referred to the clock with a precision of about 0.0005. The comparison between clocks at distant stations involves a small uncertainty in the travel time of radio signals. The total probable error of a comparison between distant clocks when the oscilloscope is used is about 0.001.

6. Instruments

A. Zenith Telescope⁸

The telescopes used by the International Latitude Service are of 4.3 in. aperture and 50 in. focal length. The tube is mounted at the end of a counterbalanced horizontal axis, which is supported by a vertical axis. The tube is free to move in the plane of the meridian, either east or west of the vertical axis, about which the telescope may be rotated. A divided circle is used for setting the telescope, but the latitude derived is dependent only upon the readings of the level and micrometer. The probable error of the latitude for one night is about $0''\cdot10$.

A large zenith telescope was developed in the U.S.S.R. for use during the International Geophysical Year. The aperture is 7.1 in. and the focal length is 93 in. Micrometer settings are made visually, but the readings of micrometer and levels are made photographically.

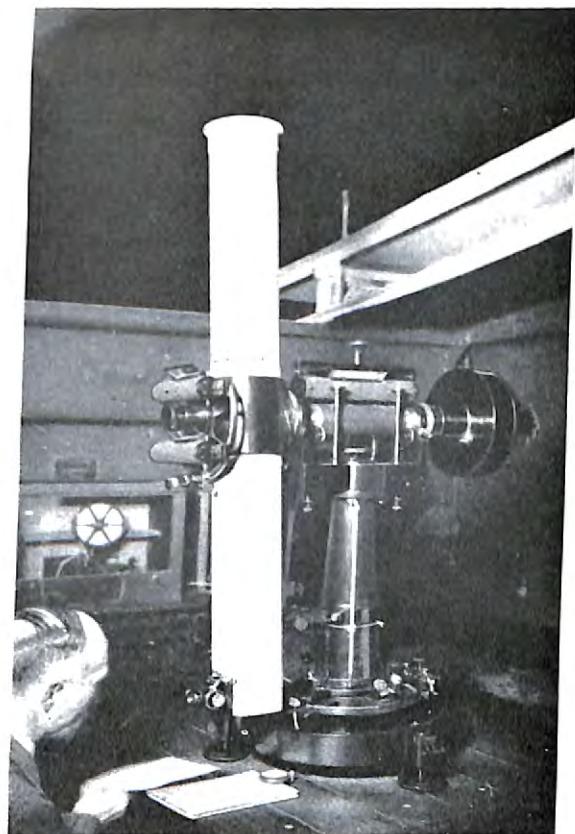


Fig. 1. Zenith telescope at International Latitude Station, Carloforte.

B. Photographic Zenith Tube⁹

The optical axis of the PZT is directed to the zenith. The rays of light from a star pass through the lens to a basin of mercury, are reflected, and come to a focus on a photographic plate which is placed just below the lens. The plate is held by a plate-carriage,